Peak Internal Fields in Direct-Coupled-Cavity Filters*

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Summary—Microwave filters are limited in their power-handling capacity by high fields generated inside the filter.

Simple formulas are derived here for the peak fields inside each cavity of a direct-coupled-cavity filter at any frequency. The computed peak fields in each cavity of a three-cavity, a four-cavity, and a six-cavity filter as a function of frequency are reproduced up to several harmonics. Inside the pass band, the internal fields are generally minimum at center frequency, rising to sharp peaks just outside the pass band.

Phase characteristics were also computed, and their relation to the internal field amplitudes is explained.

Introduction

HIGH-POWER applications of microwave filters are becoming more numerous,[1–4] as the transmitter power of radars is increased. Commonly used are microwave band-pass filters, which generally take the form of direct-coupled-cavity filters. Cohn has obtained formulas for designing such filters to meet a specified maximum insertion loss over a given pass band,[5] and has also discussed general design considerations for such high-power filters.[6]

A more detailed analysis of the power-handling capacity of direct-coupled-cavity filters is presented here. Several filters were analyzed numerically, and results for a three-cavity, a four-cavity, and a six-cavity filter are presented.

Definitions

All transmission lines and all obstacles in them will be considered to have no dissipation losses. One can therefore associate a power flow with each traveling wave, the net forward power flow being simply the difference in the powers carried by the forward and backward waves. The amplitude a_i of a forward traveling wave (traveling towards the load) is defined in magnitude by[7,8]

$$|a_i|^2 = \text{power flow due to the forward traveling wave,} \qquad (1)$$

and a similar definition holds for the amplitude, b_i say, of a backward-traveling wave (traveling towards the generator).

The same peak amplitude as is produced inside a cavity by internal reflections would occur in a properly terminated transmission line without obstacles, if sufficient power were fed into it. Thus, if a_i and b_i are the forward- and backward-wave amplitudes in the ith cavity, and if a_{in} is the amplitude of the forward traveling wave at the filter input, then the *equivalent power ratio* is defined by

$$\left(\frac{|a_i| + |b_i|}{|a_{in}|}\right)^2 \qquad (2)$$

provided that a maximum of the standing wave occurs inside the cavity.

Equivalent Power Ratios in the Cavities

A direct-coupled-cavity filter is shown schematically in Fig. 1. To find the peak amplitude inside the ith section, let a_i be the forward-wave amplitude (Fig. 2) and r_i the reflection coefficient, in that cavity. Then, if unit power emerges from the filter,

$$|a_i|^2 - |a_i r_i|^2 = 1. \qquad (3)$$

Therefore

$$|a_i| = (1 - |r_i|^2)^{-1/2}. \qquad (4)$$

Therefore, the peak internal amplitude is

$$|a_i|(1 + |r_i|) = \left(\frac{1 + |r_i|}{1 - |r_i|}\right)^{1/2} = S_i^{1/2} \qquad (5)$$

times the magnitude of the wave amplitude *emerging* from the filter, where S_i is the VSWR seen in the ith section. The "equivalent power ratio" was defined by (2) in terms of the incident power. It is thus given by

$$\left.\begin{aligned}&\text{Equivalent power ratio in the } i\text{th cavity}\\ &\quad = (\text{VSWR seen in the } i\text{th cavity, } S_i)\\ &\quad \times (\text{fraction of power transmitter by the filter})\end{aligned}\right\}. \qquad (6)$$

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[1] H. A. Wheeler and H. L. Bachman, "Evacuated waveguide filter for suppressing spurious transmission from high-power S-band radar," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 154–162; January, 1959.

[2] E. N. Torgow, "Hybrid junction-cutoff waveguide filters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 163–167; January, 1959.

[3] L. Young and J. Q. Owen, "A high power diplexing filter," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 384–387; July, 1959.

[4] J. H. Vogelman, "High-power microwave rejection filter using higher-order modes," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 461–465; October, 1959.

[5] S. B. Cohn, "Direct-coupled-resonator filters," Proc. IRE, vol. 45, pp. 187–196; February, 1957.

[6] S. B. Cohn, "Design considerations for high-power microwave filters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 149–153; January, 1959.

[7] L. Young, "Transformation matrices," IRE Trans. on Circuit Theory, vol. CT-5, p. 147; June, 1958.

[8] L. Young, "Analysis of a transmission cavity wavemeter," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 436–439; July, 1960.

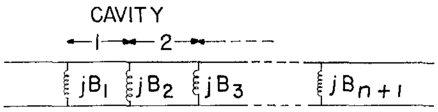

Fig. 1—Direct-coupled-cavity filter with shunt-inductive elements. $jB_1, jB_2, \cdots, jB_{n+1}$ are the normalized susceptances at band-center of the N-cavity filter.

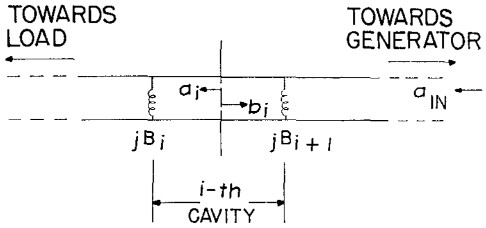

Fig. 2—Showing wave amplitudes inside the filter.

One has to be sure, of course, that the maximum of the standing wave in each cavity occurs at a real position inside the cavity, and not at a virtual position beyond the cavity in question. In the case of cavities a half-wavelength long or longer, this is always true. It also holds at least near band-center for quarter-wave transformers, and for the electric field in shunt-inductively coupled cavities.

For waveguide filters with shunt-inductive irises, and for coaxial line filters with shunt-inductive posts, the reactance of the shunt-coupling element is approximately proportional to frequency, *i.e.*, it behaves as an inductor. A digital computer program for computing the VSWR of such filters was available. By feeding in (for instance) only the first four cavities of a six-cavity filter, the VSWR in the fifth cavity was computed. The fraction of power transmitted by the whole filter was also computed by the same program applied to the whole filter; and the equivalent power ratio in the fifth (or any other) cavity was then determined from (6).

Relation between Equivalent Power Ratio and Phase

A direct-coupled-cavity filter resembles a periodic transmission system,[9,10] in which the group velocity v_g is given by

$$v_g = \frac{d\omega}{d\beta}, \tag{7}$$

where ω = radian frequency, and β is the phase constant. The phase change ϕ through a long section of length L is then $\phi = \beta L$. For our present purposes, we may write

$$v_g \propto \frac{1}{\left(\dfrac{d\phi}{d\omega}\right)}. \tag{8}$$

[9] L. Brillouin, "Wave Propagation in Periodic Structures," Dover Publications, New York, N. Y.; 1953.
[10] D. A. Watkins, "Topics in Electromagnetic Theory," John Wiley and Sons, Inc., New York, N. Y.; 1958.

Since v_g is also the velocity of propagation of energy, one may write in a pass band,

$$(\text{average energy density}) \times v_g = \text{const}, \tag{9}$$

or,

$$\text{Average energy density} \propto \frac{1}{v_g} \propto \frac{d\phi}{d\omega}, \tag{10}$$

which may be replaced by

$$\text{Equivalent power ratio} \propto \frac{d\phi}{d\omega}$$
$$\propto \text{Rate of change of phase with frequency.} \tag{11}$$

Of course, this will hold only to the extent that our direct-coupled-cavity filter resembles an infinite uniform periodic structure.

The phase ϕ appearing in (11) and elsewhere above refers to the total phase shift from input to output, whereas the computer program was designed to give the excess phase shift through the filter over the phase shift through a uniform transmission line of the same length. (One of the advantages of plotting the excess phase shift is that it permits a more accurate graphical description of phase change than does a plot of total phase shift. Thus, in Fig. 5 the latter would require an ordinate range of several thousand degrees instead of the several hundred degrees which are needed to plot the excess phase shift.) To obtain ϕ from the plotted (excess) phase shift, one then has to add back $360L/\lambda_g$ degrees, where L is the length of the filter and λ_g is the guide wavelength.

Numerical Results

Three different filters were investigated. Their circuit parameters are first given:

Filter 1; Number of cavities, $n=6$:
Susceptances (see Fig. 1):

$B_1 = B_7 =$ 1.77977 at band-center
$B_2 = B_6 =$ 6.40453 at band-center
$B_3 = B_5 =$ 9.54427 at band-center
$B_4 =$ 10.15377 at band-center.

Spacings (see Fig. 1):

$\theta_1 = \theta_6 = 147.161°$ at band-center
$\theta_2 = \theta_5 = 165.411°$ at band-center
$\theta_3 = \theta_4 = 168.511°$ at band-center.

Over-all length of filter

$$= \frac{962.166}{360} \text{ wavelengths at band-center.}$$

Filter 2; n=4:

$$B_1 = B_5 = 2.84605 \text{ at band-center.}$$
$$B_2 = B_3 = B_4 = 9.90000 \text{ at band-center.}$$
$$\theta_1 = \theta_4 = 156.741^\circ \text{ at band-center.}$$
$$\theta_2 = \theta_3 = 168.579^\circ \text{ at band-center.}$$

Over-all length of filter:

$$= \frac{650.640}{360} \text{ wavelengths at band-center.}$$

Filter 3; n=3:

$$B_1 = B_4 = 0.50000 \text{ at band-center.}$$
$$B_2 = B_3 = 1.20000 \text{ at band-center.}$$
$$\theta_1 = \theta_3 = 112.500^\circ \text{ at band-center.}$$
$$\theta_2 = 120.964^\circ \text{ at band-center.}$$

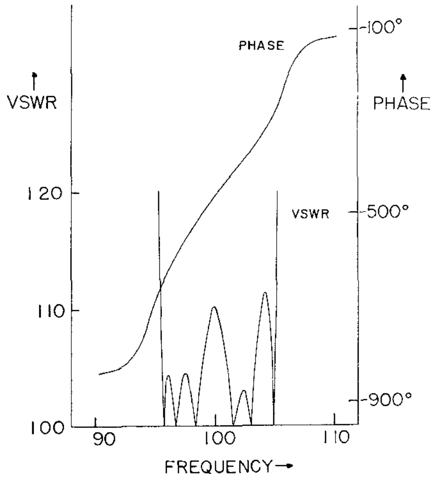

Fig. 3—VSWR and phase vs frequency of the six-cavity filter (Filter 1) over the lowest pass band.

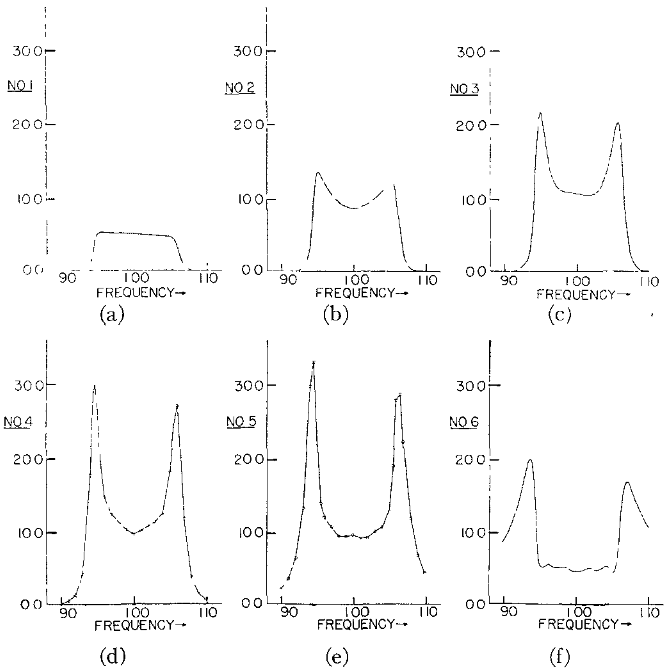

Fig. 4—Equivalent power ratios in the six cavities of Filter 1 over the lowest pass band.

Over-all length of filter

$$= \frac{345.964}{360} \text{ wavelengths at band-center.}$$

Discussion

Filter 1

The performance of Filter 1 is dealt with in Figs. 3–6. This is a six-cavity filter, the same one which is described in Fig. 9 of Cohn.[5] It has a 9.5 per cent bandwidth inside which the maximum VSWR is 1.113. This will be called its pass band. Its VSWR and (excess) phase response are plotted in Fig. 3 over and near the pass band. Note that here and throughout the graphs, the frequency axis should be labeled proportional to "reciprocal guide wavelength" in the case of dispersive waveguides.

The equivalent power ratios inside and just beyond the pass band are plotted against frequency for cavities 1 to 6 in Fig. 4(a)–4(f). By "Cavity 1" is meant the

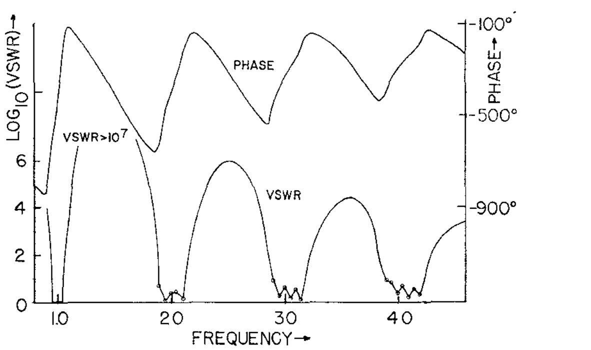

Fig. 5—VSWR and phase vs frequency of the six-cavity filter (Filter 1) up to the fourth harmonic.

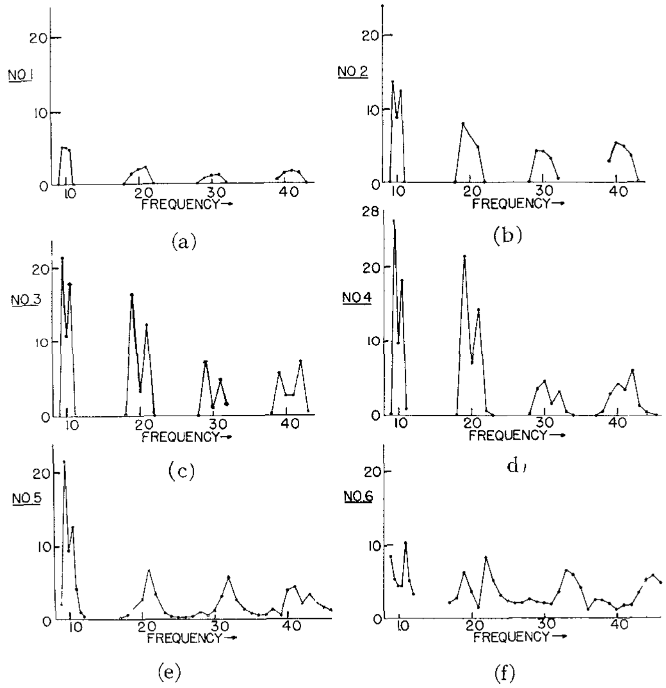

Fig. 6—Equivalent power ratios in the six cavities of Filter 1 up to the fourth harmonic.

cavity nearest the load, while "Cavity 6" is the one nearest the generator. For most of the curves, so many points were computed that a continuous curve is shown. Where only a small number of points were computed, as in Fig. 4(d) and (e), the computed points are shown by circles and joined by straight lines.

The greatest equivalent power ratio inside the pass band occurs in Cavity 4 at each edge of the band, where it reaches the value 20. This is twice as much as the equivalent power ratio in the same cavity at band-center. The greatest equivalent power ratio anywhere occurs in Cavity 5 just outside the lower end of the pass band, where it reaches at least 33.5.

The over-all length of the filter, *i.e.*, the sum of the six cavity lengths, is 962.166° at band-center. An unloaded uniform transmission line of this length increases its electrical length by 9.62° whenever the frequency increases by 1 per cent of the center frequency. This has to be added to the slope of the (excess) phase shift vs frequency curve plotted in Fig. 3 to get the total rate of change of phase with frequency, which, according to (11), should be approximately proportional to the equivalent power ratio in any cavity. It is clear by visual inspection that the slope of the phase plot in Fig. 3 does indeed rise and fall as the equivalent powers in all cavities except the first [Fig. 4(b)–(f)]. More precisely, if the slopes are determined from Fig. 3, and 9.62° per 0.01 change in frequency is added, then the respective slopes of the total phase shift at $f=0.947$, 1.00, and 1.06, where they are maximum, minimum, and maximum in turn, are in the ratio 2.2:1:1.8, which is in fair agreement with (11) and Fig. 4.

Figs. 5 and 6 represent the same six-cavity filter, plotted from below the first pass band to beyond the fourth harmonic. The same double humps in the equivalent power ratios are observed, but they get more "blurred" as the frequency and cavity number increase. The (excess) phase vs frequency slope in the graphs sometimes appears negative because, as explained before, it has been computed as a phase shifter; when the phase slope of 9.62 degrees per 0.01 increase in frequency is added, the curve always takes on a positive slope, with a staircase appearance.

Filter 2

This is a four-cavity filter, and is based on another one of Cohn's examples.[6] This one was selected to have $g_1=g_2=g_3=g_4=1$, which maximizes the power handling capacity for a given degree of selectivity.[6] Unfortunately, the VSWR inside the pass band is not very good (Fig. 7). Here $g_0=0.1$ is used to define the filter. (Cohn's filter is defined differently; it corresponds to $g_0=0.08$.) The phase vs frequency curve (Fig. 7) is remarkably straight over most of the band, and the equivalent power ratio in most cavities stays relatively constant over the same portion of the band, as would be expected from (11). There are the same two familiar sharp peaks of equivalent power ratio just outside the band, corresponding to the two kinks in the phase curve. Compare Figs. 7 and 8(a)–(d).

Fig. 4 of Cohn[6] plotted the relative electric field strength against the frequency parameter ω' of the prototype filter, taking it from $\omega'=0$ to $\omega'=1.1$. This corresponds approximately to the range $f=0.97$ to $f=1.03$ in our frequency scale.[11] The twin peaks of the equivalent power ratio against frequency curves occur at the very edges of the pass band, at approximately $f=0.95$ and $f=1.05$, and so are beyond the frequency range covered in Cohn's Fig. 4.

[11] Dr. Cohn has pointed out to the author that in Fig. 4 in "Design considerations for high power microwave filters" (*op. cit.*), Curves 3 and 4 were inadvertently interchanged. The agreement with Fig. 8 here is then quite close.

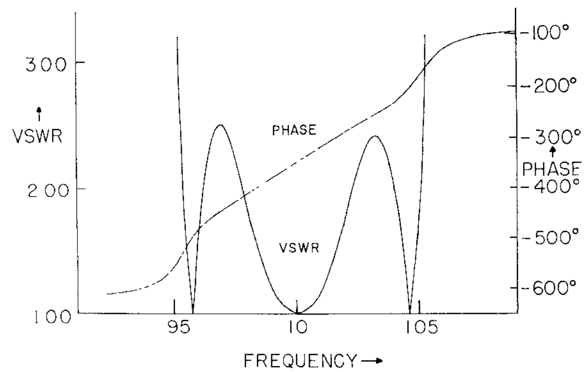

Fig. 7—VSWR and phase vs frequency of the four-cavity filter (Filter 2) over the lowest pass band.

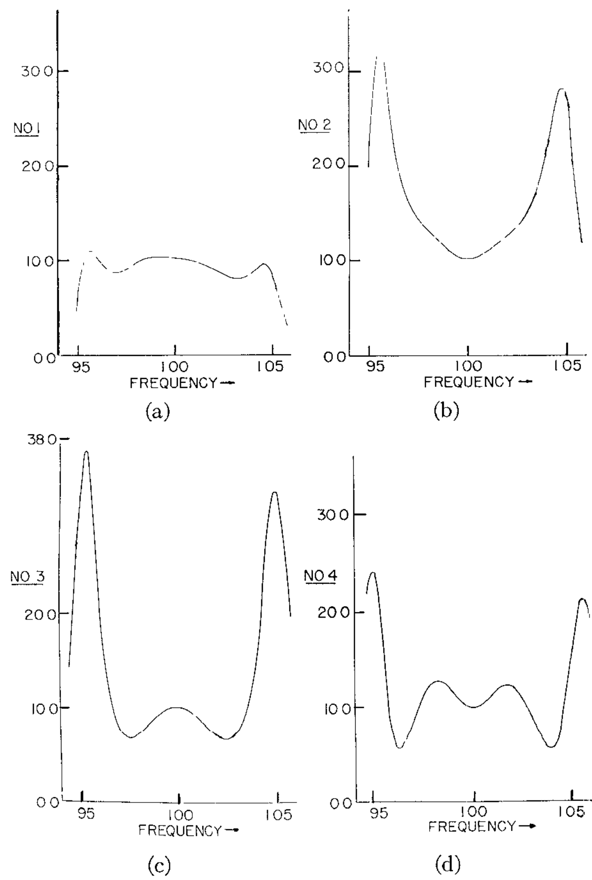

Fig. 8—Equivalent power ratios in the four cavities of Filter 2 over the lowest pass band.

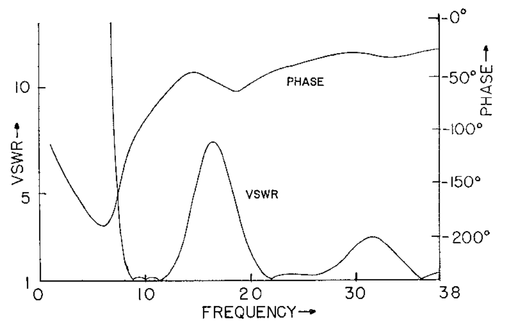

Fig. 9—VSWR and phase vs frequency of the three-cavity filter (Filter 3) up to the third harmonic.

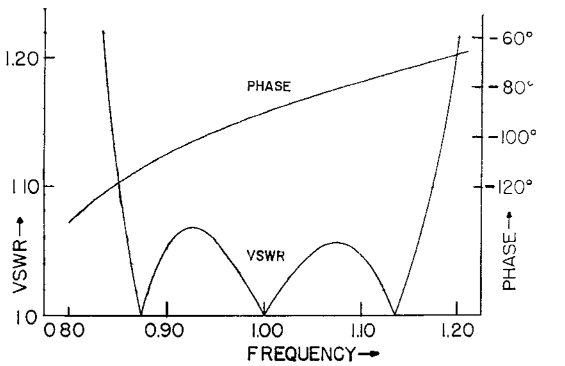

Fig. 10—VSWR and phase vs frequency of the three-cavity filter (Filter 3) over the lowest pass band.

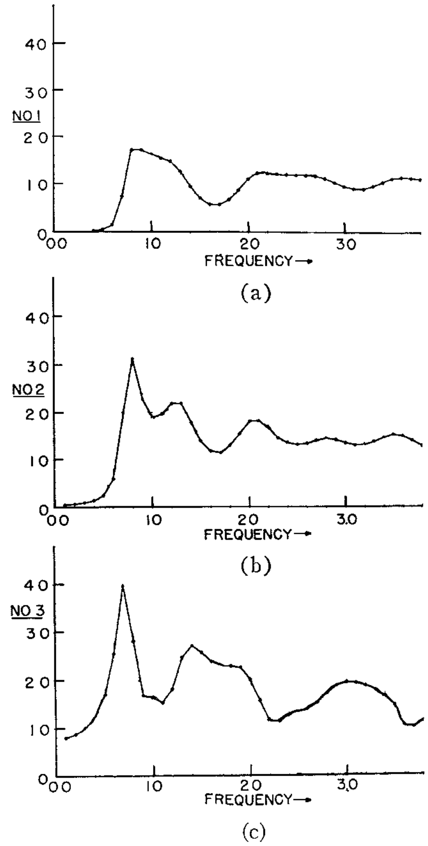

Fig. 11—Equivalent power ratios in the three cavities of Filter 3 up to the third harmonic.

Filter 3

This filter[12] has three cavities. They are much more strongly coupled than in the previous two examples. The highest VSWR above the pass band is only 7.4 (Fig. 9). It has a 30.6 per cent pass band, inside which the VSWR never exceeds 1.07 (Fig. 10). The phase plots are also given in Figs. 9 and 10. The equivalent power ratios in each cavity are plotted beyond the third harmonic in Fig. 11(a)–11(c). The general behavior is as noted in the previous two examples.

[12] L. Young, "The quarter-wave transformer prototype circuit," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 483–489; September, 1960.

Acknowledgment

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